

All-Versus-Nothing Violation of Local Realism by Two-Photon, Four-Dimensional Entanglement

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We develop and exploit a source of two-photon four-dimensional entanglement to report the first two-particle all-versus-nothing test of local realism with a linear optics setup, but without resorting to a non-contextuality assumption. Our experimental results are in well agreement with quantum mechanics while in extreme contradiction with local realism. Potential applications of our experiment are briefly discussed.

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Bell's theorem [1] resolves the Einstein-Podolsky-Rosen (EPR) paradox [2]. Arguably, it shows the most radical departure of quantum mechanics (QM) from classical intuition. It states that certain statistical correlations predicted by QM for measurements on (originally) two-qubit ensembles cannot be understood within a realistic picture, based on local properties of each individual particle. However, Bell's inequalities are not violated by perfect two-qubit correlations. Strikingly, one also has Bell's theorem without inequalities for multi-qubit Greenberger-Horne-Zeilinger (GHZ) states [3, 4]. The contradiction between QM and local realism (LR) arises for definite predictions. LR can thus, in theory, be falsified in a single run of a certain measurement. This is often called as the "all-versus-nothing" (AVN) proof [4] of Bell's theorem. Since the GHZ contradiction pertains to definite predictions, and for all systems, the GHZ theorem represents the strongest conflict between QM and LR. Further, since it involves perfect correlations, it *directly* shows that the (based on such correlations) concept of elements of reality, the missing factor in QM according to EPR, is self-contradictory.

The original GHZ reasoning is for at least three particles and three separated observers. One may ask: Can the conflict between QM and LR arise for two-particle systems, for the definite predictions, and for the whole ensemble? Namely, can the GHZ reasoning be reduced to a two-party (thus two space-like separated regions) case while its AVN feature is still retained? If so, one can then refute LR in the simplest and the most essential (i.e., unreducible) way. Further, since the EPR reasoning involved only two particles, such a refutation would be even more direct counterargument against the EPR ideas than the three-particle one. In a recent exciting debate [5, 6, 7, 8, 9] it has been shown that an AVN violation of LR does exist for two-particle four-dimensional entangled systems [9]. In this new refutation of LR, one recovers EPR's original situation of two-party perfect correlation,

but now with much less complexity. This refutation of LR becomes possible only after introducing a completely new concept [9] to define local elements of reality (LERs). The work in Refs. [5, 6, 7, 8, 9] thus demolishes the original EPR reasoning at the very outset. Here we report the first two-party AVN test of LR by developing and exploiting a source which produces a two-photon state entangled both in polarization and in spatial degrees of freedom.

The experimental setups to generate (Fig. 1a) and to measure (Fig. 1b) pairs of polarization and path entangled photons are shown in Fig. 1. A pump pulse passing through a BBO (β -barium borate) crystal can spontaneously create, with a small probability, via the parametric down-conversion [10], polarization-entangled photon pairs in the spatial (path) modes L_A and R_B . For definiteness, we prepare the entangled photon pairs to be in the maximally entangled state of polarizations $|\Psi^-\rangle_{pol} = \frac{1}{\sqrt{2}}(|H\rangle_A |V\rangle_B - |V\rangle_A |H\rangle_B)$, where $|H\rangle$ ($|V\rangle$) stands for photons with horizontal (vertical) polarization. Now if the pump is reflected through the crystal a second time, then there is another possibility for producing entangled pairs of photons again in $|\Psi^-\rangle_{pol}$, but now into the other two path modes R_A and L_B . Both pair-creation probabilities can made be equal by adjusting the foci and location of the focusing lens F. The two possible ways of producing the (polarization) entangled photon pairs may interfere: If there is perfect temporal overlap of modes R_A and L_A and of modes R_B and L_B , the path state of the pairs is $|\Psi^-(\phi)\rangle_{path} = \frac{1}{\sqrt{2}}(|R\rangle_A |L\rangle_B - e^{i\phi} |L\rangle_A |R\rangle_B)$, which is also maximally entangled. Here the two orthonormal kets $|L\rangle$ and $|R\rangle$ denote the two path states of photons. By properly adjusting the distance between the mirror and the crystal, so that $\phi = 0$, the setup in Fig. 1a generates the state [9, 11] $|\Psi\rangle = |\Psi^-\rangle_{pol} \otimes |\Psi^-(0)\rangle_{path}$, which is exactly the desired maximally entangled state in both polarization and path. Actually $|\Psi\rangle$ can also be

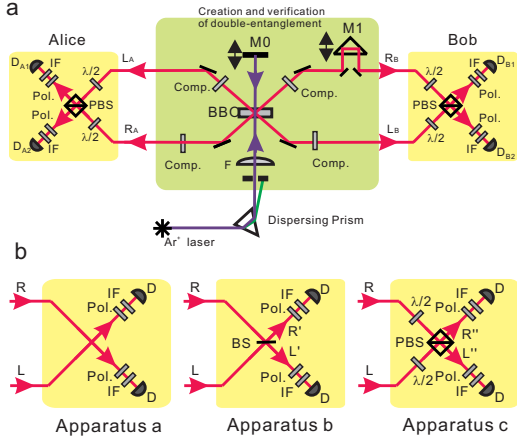


FIG. 1: Experimental Setups. (a) An ultraviolet beam from Argon ion laser (351.1 nm, 120 mW) is directed into the BBO crystal from opposite directions, thus can create photon pairs (with wave length 702.2 nm) in $|\Psi\rangle$. Four compensators (Comp.) are used to offset the birefringent effect caused by the BBO crystal during parametric down-conversion. The reflection mirrors M0 and M1 are mounted on translation stages, to balance each arm of the interferometer and to optimize the entanglement in path. (b) Apparatuses to measure all necessary observables of doubly entangled states. D is single-photon count module, with collection and detection efficiency 26%; IF is interference filter with a bandwidth of 2.88nm and a center wavelength of 702.2nm; Pol. is polarizer. Apparatus c has been included in (a) at the locations of Alice and Bob.

interpreted as a maximally entangled state of two four-dimensional subsystems in a $4 \otimes 4$ dimensional Hilbert space [9]. Figure 2 shows how to achieve good temporal overlap of modes R_A and L_A and of modes R_B and L_B and to adjust the phase $\phi = 0$.

Then photon- A and photon- B are, respectively, sent to Alice and Bob (actually the two observation stations are about 1 meter apart in our experiment). We emphasize that $|\Psi\rangle$ indeed corresponds to the case where there is one and only one pair production after the pump pulse passes twice through the BBO crystal. We observed about 3.2×10^4 doubly-entangled photon pairs per second.

One can define the following set of local observables to be measured by Alice and Bob: $|H\rangle\langle H| - |V\rangle\langle V| \equiv z$ and $|+\rangle_{pol}\langle +| - |-\rangle_{pol}\langle -| \equiv x$ ($|R\rangle\langle R| - |L\rangle\langle L| \equiv z'$ and $|+\rangle_{path}\langle +| - |-\rangle_{path}\langle -| \equiv x'$) are two Pauli-type operators for the polarization (path) degree of freedom of photons. Here $|\pm\rangle_{pol} = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)$ and $|\pm\rangle_{path} = \frac{1}{\sqrt{2}}(|R\rangle \pm |L\rangle)$. Further on, Alice's observables will be specified by subscript A and Bob's by subscript B .

According to Ref. [9] the six local operators $z_A, z'_A, x_A, x'_A, z_A z'_A$, and $x_A x'_A$ for Alice ($z_B, z'_B, x_B, x'_B, z_B z'_B$, and $x_B x'_B$ for Bob) can be utilized to define the LERs for the two-party system. This is due to the fact that for the two photons described by $|\Psi\rangle$ QM makes the

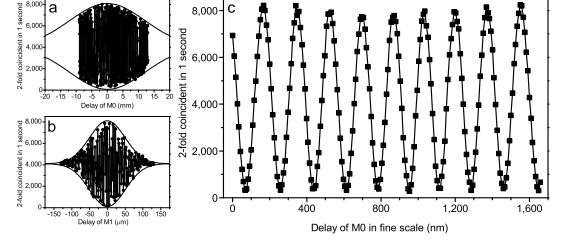


FIG. 2: Interference fringe observed when M0 being moved to achieve perfect temporal overlap and to adjust the phase $\phi = 0$. (a,b) Observed interference fringe for roughly achieving the temporal overlap of modes R_A and L_A and of modes R_B and L_B . We measure the twofold coincidence between the output modes toward detectors D_{A2} and D_{B2} behind 45° polarizers, by scanning the position of M0 with a step sizes of 1 mm (a) and of 1 μm (b). The envelope of the observed twofold coincidence varies indicating the visibility of the two-photon coherence. Inside the coherent region, the best visibility is obtained at the position where perfect temporal overlap is achieved. We perform fine adjustment of the position of M1 and repeat the scanning of M0 until the best visibility is obtained. (c) We use a piezo translation stage to move the mirror M0 to perform a fine scan around the centre of the envelope, with a step size of 1.6 nm. By setting the piezo system to a position where we observe maximum twofold coincidence of D_{A2} - D_{B2} , we then achieve $\phi = 0$.

following predictions:

$$z_A \cdot z_B |\Psi\rangle = -|\Psi\rangle, \quad z'_A \cdot z'_B |\Psi\rangle = -|\Psi\rangle, \quad (1)$$

$$x_A \cdot x_B |\Psi\rangle = -|\Psi\rangle, \quad x'_A \cdot x'_B |\Psi\rangle = -|\Psi\rangle, \quad (2)$$

$$z_A z'_A \cdot z_B \cdot z'_B |\Psi\rangle = |\Psi\rangle, \quad x_A x'_A \cdot x_B \cdot x'_B |\Psi\rangle = |\Psi\rangle, \quad (3)$$

$$z_A \cdot x'_A \cdot z_B x'_B |\Psi\rangle = |\Psi\rangle, \quad x_A \cdot z'_A \cdot x_B z'_B |\Psi\rangle = |\Psi\rangle, \quad (4)$$

$$z_A z'_A \cdot x_A x'_A \cdot z_B x'_B \cdot x_B z'_B |\Psi\rangle = -|\Psi\rangle. \quad (5)$$

Now the nine local variables for Alice will be arranged into three groups/devices: $a_A = (z'_A, x_A, x_A \cdot z'_A)$, $b_A = (z_A, x'_A, z_A \cdot x'_A)$ and $c_A = (z_A z'_A, x_A x'_A, z_A z'_A \cdot x_A x'_A)$, while for Bob $a_B = (z_B, z'_B, z_B \cdot z'_B)$, $b_B = (x_B, x'_B, x_B \cdot x'_B)$ and $c_B = (z_B z'_B, x_B x'_B, z_B z'_B \cdot x_B x'_B)$. For each operational situation (e.g., a_A) the observer receives two-bit readouts (results). The bit values, because of the use of product variables, are here denoted as ± 1 , instead of 0 and 1. In the case of the device a_A Alice can read out x_A and z'_A , and by multiplication get $x_A \cdot z'_A$. With b_A she can measure the values of z_A and x'_A and therefore fix the derivative value of their product $z_A \cdot x'_A$. Finally if her choice is c_A she gets $z_A z'_A$ and $x_A x'_A$ and their algebraic product $z_A z'_A \cdot x_A x'_A$. It is important to note that the last value is not operationally equivalent to $z_A \cdot z'_A \cdot x_A \cdot x'_A$, and that it is impossible to mea-

sure all these values in the product for a single system. Similarly, Bob can choose between three operational situations, namely a_B via which he gets the access to z_B and z'_B and their product $z_B \cdot z'_B$, b_B which gives x_B , x'_B and $x_B \cdot x'_B$, and finally c_B producing $z_B x'_B$, $x_B z'_B$ and $z_B x'_B \cdot x_B z'_B$. If the above measurements are performed in spacelike separated regions, then by Einstein's locality, any measurement performed on one photon would not in any way disturb actions on, and results for, the other photon. Following EPR, the perfect correlations in Eqs. (1)-(5) allow a local realistic interpretation by assigning pre-existing measurement values to operators or operator products that are separated by (\cdot) . These values [6, 9] would be EPR's elements of reality. Each operational situation for Bob can be used to establish the EPR elements of reality for three of Alice's variables. And since we have listed nine perfect correlations, three for each operational situation at Alice's side, all the above listed variables of Alice seemingly, according to EPR, can be associated with elements of reality. The same holds for Bob's variables.

However the above system of LERs turns out to be inconsistent. Let $m(\Lambda)$ stand for the LER associated with the variable Λ . If the quantum perfect correlations are to be reproduced, the following relations between the LERs must hold:

$$m(z_A)m(z_B) = -1, \quad m(z'_A)m(z'_B) = -1, \quad (6)$$

$$m(x_A)m(x_B) = -1, \quad m(x'_A)m(x'_B) = -1, \quad (7)$$

$$m(z_A z'_A)m(z_B)m(z'_B) = 1, \quad (8)$$

$$m(x_A x'_A)m(x_B)m(x'_B) = 1, \quad (9)$$

$$m(z_A)m(x'_A)m(z_B x'_B) = 1, \quad (10)$$

$$m(x_A)m(z'_A)m(x_B z'_B) = 1, \quad (11)$$

$$m(z_A z'_A)m(x_A x'_A)m(z_B x'_B)m(x_B z'_B) = -1. \quad (12)$$

Note that the values of, say z_A and z'_A , are defined in different operational situations, namely b_A and a_A , while the value of the variable $z_A z'_A$ is obtainable operationally in situation c_A . This dispels the possible fear of the reader that a local non-contextuality assumption is tacitly used here - *nowhere do we assume that* $m(z_A z'_A) = m(z_A)m(z'_A)$, etc. Note that for all our variables $m(\Lambda) = \pm 1$. If one multiplies any subset of the eight equalities from the above set, side by side, then as a result one gets the ninth one, but with a wrong sign. That is, if LR holds, and the LERs satisfy eight of the above relations, which they must, if they are to reproduce the eight quantum predictions, then on the level of gedanken experiment, LR predicts that every measurement of the LERs related with the ninth equation must give a perfect correlation of local results, which, however is perfectly opposite to the quantum prediction. Outcomes predicted to definitely occur by LR are never allowed to occur by QM and vice versa. Thus, one indeed has an AVN conflict between LR and QM.

Importantly, Ref. [9] also provided a linear optics implementation of the above experiments, where both Alice and Bob need to measure nine local variables arranged in three different operational situations: a_A , b_A and c_A for Alice and a_B , b_B and c_B for Bob. Figure 1b shows the devices for measuring all the above local observables: Apparatus a (b) measures the variables in a_A and a_B (b_A and b_B). By adjusting the polarizers along the two paths, one can measure the polarization in either $|H/V\rangle$ or $|\pm\rangle_{pol}$ basis. The measurements in the $|\pm\rangle_{path}$ basis can be achieved by interfering the two paths at a beam splitter (BS) which affects the transformations $|R\rangle \rightarrow |+\rangle_{path}$ and $|L\rangle \rightarrow |-\rangle_{path}$.

Apparatus c in Fig. 1b measures simultaneously the variables in c_A or c_B , where the observables contain always the polarization and the path information simultaneously. Let us first consider measuring the former. Note that a polarizing BS (PBS) transmits horizontal and reflects vertical polarization. If the optical axes of the two half-wave plates ($\lambda/2$; HWP) in apparatus c are horizontal, the polarizations of the photons will not be affected after passing through the HWP. Then for Alice's apparatus c, the outgoing port R'' (L'') of the PBS corresponds to the case of $z_A z'_A = +1$ ($z_A z'_A = -1$). For example, an H -polarization photon from L -path will appear at the R'' -port of the PBS, with the result $z_A = +1$, $z'_A = -1$ and $z_A z'_A = -1$. At the same time, a V -polarization photon from R -path will appear at the same R'' -port of the PBS, with the result $z_A = -1$, $z'_A = +1$ and $z_A z'_A = -1$. Due to the fact that the photon in both path modes leaves the PBS simultaneously into R'' -path, the information on whether the photon was transmitted or reflected will be erased if one measures the photon polarization in the $|\pm\rangle_{pol}$ basis along the R'' -path. After such an information erasure, one can find that the case of $x_A x'_A = +1$ ($x_A x'_A = -1$) corresponds to the photon in $|+\rangle_{pol}$ polarization ($|-\rangle_{pol}$ polarization). Thus, by choosing appropriate polarizers, apparatus c can then measure the variables in c_A simultaneously. For Bob's apparatus c, the only difference stems from the two HWP, which now affect the transformations $|H\rangle \rightarrow |+\rangle_{pol}$ and $|V\rangle \rightarrow |-\rangle_{pol}$. Following an argument similar to Alice's apparatus c, one sees that Bob's apparatus c measures $z_B x'_B$ and $x_B z'_B$ simultaneously and thus also gives the result of $z_B x'_B \cdot x_B z'_B$ at the same time.

As we argued above, the operators or operator products separated by (\cdot) can be identified as EPR's elements of reality. The non-contextuality assumption is not used if the three variables of each group are measured by one and the same linear optical device [7, 9].

The measured results are consistent with the QM predictions for $|\Psi\rangle$ with high visibilities: $E(z_A \cdot z_B) = -0.98526 \pm 0.00094$, $E(z'_A \cdot z'_B) = -0.99571 \pm 0.00032$, $E(x_A \cdot x_B) = -0.98572 \pm 0.00092$, $E(x'_A \cdot x'_B) = -0.92999 \pm 0.00200$, $E(z_A z'_A \cdot z_B \cdot z'_B) = 0.98538 \pm 0.00094$, $E(x_A x'_A \cdot x_B \cdot x'_B) = 0.88037 \pm 0.00296$, $E(z_A \cdot x'_A \cdot z_B \cdot x'_B) =$

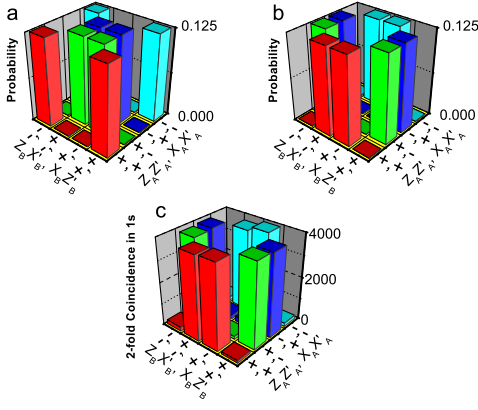


FIG. 3: Predictions of LR (a) and of QM (b), and observed results (c) for the $z_A z'_A \cdot x_A x'_A \cdot z_B z'_B \cdot x_B z'_B$ experiment.

0.90254 ± 0.00269 , and $E(x_A \cdot z'_A \cdot x_B z'_B) = 0.98560 \pm 0.00092$. Here the correlation functions $E(p) = [C(p = +1) - C(p = -1)]/[C(p = +1) + C(p = -1)]$, where $C(p = \pm 1)$ are the counting numbers when the measured variable $p = \pm 1$. Each of the above data was collected within one second by using apparatus a, b or c in Fig. 1b.

Once the perfect correlations of $|\Psi\rangle$ were closely reproduced in the measurement, we performed the $z_A z'_A \cdot x_A x'_A \cdot z_B z'_B \cdot x_B z'_B$ ($\equiv \mathcal{M}$) experiment, for which QM and LR predict opposite results (Fig. 3a and 3b). The measured result of \mathcal{M} is shown in Fig. 3c. With a fidelity of about 96% only those events predicted by QM were observed in our experiment. This amounts to a very high precision experimental realization of the first two-photon AVN test of LR.

The AVN argument against LR in Ref. [9] is based on experimentally unachievable perfect correlations. We also observed spurious events, although not too often. For instance, although an extremely high fidelity of 96% has been achieved in the \mathcal{M} experiment, there is still about 4% of detected events, that is in agreement with LR. Thus, for our argument to hold, we can assume that these spurious events are only due to experimental imperfections. Note that, the spurious events are mainly due to the imperfection of parametric down-conversion source and the limited interference visibility on the BS and PBS. Alternatively, a Bell-type inequality in Refs. [6, 9] can be used. For any LR model one has $\langle \mathcal{O} \rangle_{LRT} \leq 7$, where $\mathcal{O} = -z_A \cdot z_B - z'_A \cdot z'_B - x_A \cdot x_B - x'_A \cdot x'_B + z_A z'_A \cdot z_B \cdot z'_B + x_A x'_A \cdot x_B \cdot x'_B + z_A \cdot x'_A \cdot z_B z'_B + x_A \cdot z'_A \cdot x_B z'_B - \mathcal{M}$. The observed value for \mathcal{O} is 8.56904 ± 0.00533 , which is a violation by about 294 standard deviations.

To summarize, with an unprecedented visibility of 95% (i.e. the average of the absolute value of nine correlation functions observed) we have reported the first experimental AVN falsification of LR using the two-photon

four-dimensional entanglement. In contrast to previous GHZ experiments [12, 13], our experiment does not require any post-selection. This allows an immediate experimental verification of a quantum pseudo-telepathy game [14]. The high-quality double entanglement also enables to implement deterministic and highly-efficient quantum cryptography [15] based on the tested AVN falsification of LR. Of course, as in almost all of the existing experiments testing LR, our experiment also has certain well known loopholes, such as the locality and efficiency loopholes. Finally, the full usage of the interference in paths of photons enables one to entangle two photons in Hilbert space of arbitrarily high dimensions in a way that is easier than entangling two photons in their orbital angular momentum states [16]. Such hyperentanglement and its manipulation [17] may be useful in some quantum cryptography protocols [18] and in test of Bell's inequalities for high-dimensional systems [19].

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